Measurements of BMR, AMR, and RQ for the individual astronauts are most easily made in zero-g by means of a modified Douglas-bag technique,[2] which employs lightweight meteorological balloons attached to high-velocity, low-resistance mouthpieces and valve assemblies. It is assumed that a gas chromatograph will be on board the space station for the accurate analysis of trace contaminants, atmospheric composition, and respiratory gases. Preliminary experimental results are available and will be reported by the present and other authors elsewhere.

Estimated Weights for 30-, 60-, and 100-Day Missions

The design criteria and inputs given in Tables 1–3,6, and 7 for the LSS just described have been used to estimate the curves for total LSS weight vs crew size for 30-, 60-, and 100-day missions in Fig. 5. These weights include the weights of required power and heat rejection equipment. Requirements for resupply for an equal time after a given period would be reduced by the fixed weight penalty of the solar power supply.

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Approach to Space Station Logistics Optimization

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The paper presents an analytic-computational method, utilizing dynamic programming techniques, designed to minimize the cost of supplying an extraterrestrial base. The minimization is accomplished by selection and scheduling of supply vehicles. The criterion of optimality was chosen to minimize the expected cost in order to account for the probabilistic nature of the problem implied by vehicle reliabilities of less than 1. Illustrative results from a current study of small space station logistics conducted by the described method are presented.

Introduction

ONE of the distinguishing features of any mission in space is its complete dependence on supplies originating on earth; satisfactory logistics planning is, therefore, central to the eventual effective exploration and utilization of space. This is particularly true of missions requiring the delivery of comparatively large cargo tonnages, as it may be the case with some extraterrestrial bases. In view of the cost of delivering a pound of supplies to even a "nearby" base such as an orbiting space station, it is clear that in many cases the very feasibility of proposed missions may well depend on one's ability to reduce logistics costs to the minimum level possible.

It is the purpose of this paper to contribute to this end by presenting an analytic-computational method well suited to determining some of the major defining elements of a logistics operation, namely, the optimal choice and scheduling of supply vehicles. The criterion of optimality is to minimize the expected cost of the supply operation; this criterion allows ready inclusion of the cost-reliability tradeoff on supply vehicle selection. The method of analysis is based on dynamic programming techniques. See, e.g., Refs. 1–4; Refs. 1–3 represent very readable and thorough presentations of the dynamic programming techniques utilized in this paper, and Ref. 4 describes a technique that will be found most useful in attempting the numerical solution of problems of higher dimensionality than discussed herein. Its application is briefly illustrated by considering the problems of supplying a manned space station.

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Statement of the Problem

The purpose of this paper is to describe and demonstrate a method for the solution of the following basic problem in supplying an extraterrestrial base as follows.

Table 1 Minimum expected cost obtainable by optimal supply vehicle selection

Station is allowed to decay							
From alt., naut mile	To altitude, naut mile (cost in millions)						
	250	225	200	175	150	125	100
300	17.05	17.05	20.29	23.71	25.50	32.82	55.85
250		20.39	23.72	25.36	27.61	32.82	57.77
225			25.37	25.44	27.61	33.33	58.08
200				25.52	27.82	33.38	60.46
175					33.22	40.13	66.47
150						41.62	73.41
125							80.11

Given

1) The logistics demand level at the base as any function of time.

2) A range of choice in supply vehicles, either in existence or merely proposed, available for logistics service.

3) The characterization of each of these vehicles in terms of: a) reliability, defined as the probability of completing the supply mission; b) payload delivery capacity; and c) effective cost in terms of dollars per pound delivered.

4) The cost, as any function of projected base life remaining, of any emergency action (say, rescue or abort) necessitated by a succession of supply vehicle failures and resultant low supply levels at the base.

5) Constraints established for convenience or safety, such as: a) minimum supply levels at the base considered to be safe; b) the nature of the action to be taken (i.e., rescue-abort operations) if supply levels should sink to the safe limit; and c) minimum reliability requirements on launches of cargoes deemed particularly valuable.

Required

1) The minimum expected cost of supplying the base, with "expected cost" defined in the accepted mathematical sense of "expected value."

2) The vehicle selection that will yield this minimum cost.

3) The minimum expected cost and optimal vehicle choice as a function of: a) duration of the remaining part of the supply operation; and b) amount of supplies on hand at the base.

4) The variation of these results with "given" information.

Major Assumptions

1) Only one supply vehicle may be launched at a time.

2) A vehicle is not selected or launched until the success or failure of the previous vehicle has been established.

3) The component items of the cargo to be delivered are not distinguished from each other.

4) Vehicle cost and reliability are invariant with the number of launches.

It is to be emphasized that these simplifying assumptions have been made for the purpose of presenting more clearly the main outlines of the dynamic programming approach to vehicle selection. This approach is a practical one without these assumptions. In point of fact, they have been eliminated from a more advanced statement of the problem now undergoing numerical analysis.

Statement of the Solution

To initiate the solution of the problem stated in the second section of this paper, the following are defined:

W = supply level on board at the first of N remaining periods, lb

W_m = minimum supply level acceptable at any time, lb

$f_N(W)$ = expected minimum cost of supplying the station over N remaining time periods if an optimal launch schedule and an optimal vehicle selection are used, subject to stated constraints, dollars

$b(N)$ = consumption during the first of the N remaining time periods, lb

$P_m(N)$ = minimum acceptable reliability for a supply vehicle to be considered for launch during the first of N remaining periods

$A(N)$ = cost of action necessitated by supplies falling to or below minimum safe levels, i.e., cost of rescue-abort operations, with N periods to go, dollars

J = decision variable
= -1 if station is to be aborted
= 0 if no launch is made
= 1, 2 . . . if vehicles 1, 2 . . . are to be used

X_j = delivery capability of vehicle J, lb

C_j = delivered cost for vehicle J, dollars/lb

P_j = probability of vehicle J accomplishing its mission

The following constraints limit the free choice of a J if at any time one or both of the following situations obtains: a) the supply level has fallen below W_m; or b) it will fall below W_m if the next launch fails. Then the base is abandoned, incurring a dollar cost of $A(N)$, unless: a) a vehicle is available with sufficient payload capability to immediately re-establish a safe supply level; b) this vehicle satisfies minimum reliability criteria; and c) it is cheaper to maintain the base than to abandon it.

The problem is solved when $f_N(W)$ and J are determined for all N and W. As the first step, $f_1(W)$ is determined in the following manner:

If

$$W - b(1) \geq W_m \tag{1}$$

then

$$f_1(W) = 0 \qquad \text{and} \qquad J = 0 \tag{2}$$

If

$$W - b(1) < W_m \tag{3}$$

then

$$f_1(W) = A(1) \qquad \text{and} \qquad J = -1 \tag{4}$$

unless a J can be found to satisfy both

$$W - b(1) + X_j \geq W_m \tag{5}$$

and

$$P_j \geq P_m(1) \tag{6}$$

with the search extending over $J = 1, 2 \ldots J_{\max}$.

If the J satisfying (5) and (6) are denoted by $J^* = J_1, J_2, \ldots J_k$, then

$$f_1(W) = \min \begin{bmatrix} J = -1: \; A(1) \\ J \neq -1: \; \overset{\min}{J^*} \; [C_jX_j + (1 - P_j)A(1)] \end{bmatrix} \tag{7}$$

If two or more J satisfy (7), then of these J the one for which P_j is highest will be considered the optimal decision; if two or more equally large P_j are selected by this process, then, in the absence of a less arbitrary criterion, the numerically largest of the J yielding this maximum P_j will be selected.

Having determined $f_1(W)$, one can now proceed with the tabulation of $f_N(W)$: If

$$W - b(N) > W_m \tag{8}$$

$$f_N(W) = \min \begin{bmatrix} J = 0\colon f_{N-1}(W - b(N)) \\ J \neq 0\colon \overset{\min}{J^*}[C_jX_j + P_jf_{N-1}(W - b(N) + X_j) + (1 - P_j)f_{N-1}(W - b(N))] \end{bmatrix} \quad (9)$$

for $N = 2, 3 \ldots N_{\max}$.

Equation (9) is a direct application of Bellman's principle of optimality: "An optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Continuing, note that the multiplication of $f_{N-1}(W)$ by the probability of the outcome considered, i.e. (P_j) or $(1 - P_j)$ is a result of optimizing "expected cost." If

$$W - b(N) < W_m \quad (10)$$

then

$$f_N(W) = A(N) \qquad \text{and} \qquad J = -1 \quad (11)$$

unless a J can be found to satisfy both

$$W - b(N) + X_j \geq W_m \quad (12)$$

and

$$P_j \geq P_m(N) \quad (13)$$

with the search extending over $J = 1, 2 \ldots J_{\max}$.

If the J satisfying (12) and (13) are denoted by $J^* = J_1, J_2 \ldots J_k$, then

$$f_N(W) = \min \begin{bmatrix} J = -1\colon\ A(N) \\ J \neq -1\colon\ \overset{\min}{J^*}\{C_jX_j + P_jf_{N-1}[W - b(N)] + (1 - P_j)A(N)\} \end{bmatrix} \quad (14)$$

If two or more J satisfy (9) or (14), then of these J the one for which P_j is highest will be considered the optimal decision; if two or more equally large P_j are selected by this process, then the numerically largest of the J yielding this maximum P_j will be retained.

The equations and inequalities (1–14) have been incorporated into an IBM 7090 FORTRAN language program to facilitate rapid determination of the J and $f_N(W)$ for all N and W of interest.

Example

Some preliminary and partial results excerpted from a study currently in progress can be used to illustrate outputs from the foregoing method. This study is concerned with determining that combination of operating altitude and supply vehicles for a manned space station which will minimize the expected cost of supplying the station for one year. The tradeoff being investigated arises from the decrease of station-keeping and attitude-control fuel requirements with an increase in the maximum altitude and the minimum altitude (the altitude to which the station is allowed to decay before being reboosted to the maximum altitude) and the individually different rate at which potential supply vehicles lose payload capability with increasing altitude. The demand for consumables such as food, water, and air is, of course, essentially invariant with altitude. Assuming that the Atlas Agena D, the C-1, the C-IB, and the Titan II are available and modified for supply service, the problem is to determine the optimum combination of altitudes and supply vehicles.

Basing calculations on the requirements of a four-man station, the minimum expected cost of the logistics operation, using optimal vehicle selection, is given by Table 1 as a function of minimum and maximum station orbital altitudes. The optimal vehicle selection is shown in Table 2, also as a function of altitudes. Inspection of these tables suggests the following general trends:

1) The maximum orbital altitude and the minimum orbital altitude should both be as high as possible; in other words, within the range of altitudes investigated, the station should be as high as possible and should be allowed to decay as little as possible if the cost of the supply operation is to be minimized.

2) Vehicles with relatively low payload capabilities, such as Atlas Agena D and Titan II, are optimal at higher altitudes; at intermediate altitudes high payload vehicles, such as C-1 and C-IB, appear optimal. Low altitudes seem to require mixed vehicle inventories.

From the point of view of resupply economics the best decision is, therefore, to place the station into an orbit with a 300-naut mile maximum and to let it decay as little as possible. If it is allowed to decay to 250 naut miles, the minimum expected cost of supplying it with fuel and consumables is 17.05 million dollars per year and the optimal vehicle selection is two Atlas Agena D's per year.

Very clearly, whether the suggested method or any other technique is employed, conclusions such as the foregoing will greatly depend on one's estimate of the supply requirements, vehicle costs, performance, reliability, and other input data. The generality of these conclusions and their applicability to a specific situation the reader may have in mind also depends, of course, on the fidelity with which the mathematical model previously presented approximates the situation of interest.

For these reasons, it is emphasized that the purpose of this section is to present an example of the results to be expected from an application of the method of analysis presented in this paper; it is not intended to arrive at general conclusions and recommendations with respect to the selection of resupply vehicles and space station operating altitudes.

Some Comments on the Solution

The objective is to minimize the expected cost of a logistics operation; the method attempts this by optimizing over the choice of supply vehicles. It is obvious that the cost of the logistics operation is a function of many other variables besides vehicle selection and that for a complete solution they, too, should be included in the optimization. The purpose of the following remarks is to comment on this and related problems.

First, it is necessary to observe that logistics problem variables may be divided into two sets. The first set contains those parameters that contribute to setting the supply requirements of the station and may be exemplified by crew size, station altitude, attitude control system used, station-keeping propulsion system used, and the configuration, shape, and resulting drag of the station. A variation in any member of this first set may, therefore, be expressed as a variation in the demand schedule $b(N)$.

Table 2 Optimal launch vehicle[a] selection chart[b] as a function of altitudes

	Station is allowed to decay						
From alt., naut mile	To altitude, naut mile						
	250	225	200	175	150	125	100
300	1–1	1–1	1–1	4–4	1–1–1–1	3	1–1–3–1
250		1–4	4–4	1–1–1	2	3	1–3–1–1
225			1–1–1	1–1–1	2	3	1–3–1–1
200				1–1–1	2	3	3–2
175					3	3–1	3–3
150						3–1	3–3
125							3–3–1–1

[a] Vehicles considered: 1 = Atlas Agena D; 2 = C–1; 3 = C–1B; 4 = Titan II.

[b] Use of chart: For a maximum and minimum station altitude of 200 and 175 naut miles, respectively; the optimal launch sequence is 1–1–1, i.e., 3 Atlas Agena D's in succession.

The second set defines the choices open in meeting the demand schedule set by any combination of the first set of parameters; examples are:

1) The set of vehicles from which one may make selections.

2) The payload vs altitude and inclination, effective cost and reliability characteristics of these vehicles.

3) Supplies initially launched with the station.

4) Constraints, such as: a) minimum supply level to be maintained for reasons of safety, and b) minimum reliability requirements on launches of particularly valuable cargoes.

These variables appear explicitly as inputs to the solution proposed in the preceding section. Most of the logistics problem variables are, therefore, implicitly or explicitly included in the solution if a demand schedule $b(N)$, the vehicle characteristics X_j, C_j, P_j, and other input parameters such as W_m and $P_m(N)$ are specified. Conversely, if the solution is studied parametrically with $b(N)$, X_j, and similar inputs, the solution is actually being studied as a function of variables other than vehicle choice.

A number of constraints are also implicit in the formulation or may be readily included. An example of the former is a limitation on launch rates, i.e., the number of vehicles which can be launched per month; such a limitation may be imposed by the availability of launch support facilities. This constraint is implicitly included in the foregoing model by limiting the number of vehicles which can be launched to only one per decision stage, but allowing the time duration of a decision stage to assume any value. For instance, if only one vehicle may be launched per month, the time duration of a decision stage would be one month; if one vehicle may be launched every three months, the time duration of a decision stage would be made equal to three months. To facilitate this, the consumption rate $b(N)$ was defined as the consumption per decision stage rather than as the consumption per unit time.

There may arise circumstances that would prohibit the launch of a vehicle during the time period under consideration. Such may be the case if the delivery of a full load would cause the cargo space on board the lab to be exceeded, i.e., there would be no room to store the newly arrived supplies. The model was formulated to define a system where the supplies would be stored in the supplies vehicles and not on board the space station; the constraint required is, however, readily included by disallowing the choice of all nonzero (J) unless

$$W + X_j \leq W_{\max} \tag{15}$$

is satisfied, where $(W_{\max})$ is the storage capacity of the station.

Finally, it will be of interest to comment on the growth capability of the model, specifically the elimination of the assumption that vehicle cost is invariant with the number of launches. Two modifications of the cost accounting method are required. The first of these is the incorporation of research, development, and testing costs, i.e., the costs that must be incurred before the first operational vehicle can be made available. The second modification required is a provision for allowing the unit cost of each operational vehicle to decrease with the number of vehicles produced, i.e., the incorporation of a learning curve. Only the major steps will be indicated here; a more complete treatment of the problem will be presented in a subsequent paper.

The first step is to define the following:

$T_N(L,W)$ = minimum expected resupply cost over N stages, beginning the process with vehicle combination L already developed, and using optimal vehicle selection

$V_N(L,W,J)$ = expected number of vehicles of type J which will be used over the N remaining stages, beginning the process with vehicle combination L already developed, and using optimal vehicle selection

Any particular value of L represents a unique combination of resupply vehicles for which all fixed costs have already been paid. For instance, $L = 5$ may represent a state of affairs in which vehicles of type A and B have been developed, but type C has not; $L = 6$ may represent a state in whch all three have been developed. The quantity L is thus a state vector component. A change in L may, therefore, require additional expenditures, which may be tabulated by the following function: $S(L,K)$ = cost of transforming from L to K, i.e., the additional cost of making vehicles available which are included in K but not in L. Further defining:

C_j = unit cost of a vehicle of type J
X_j = payload of a vehicle of type J
P_j = reliability of a vehicle of type J
b = consumption rate, i.e., decrease in W per stage
α_j = $W - b + X_j$
β = $W - b$

one may write

$$T_N(L, W) = \overset{\min}{J} [C_j + S(L,K) + P_j T_{N-1}(K,\alpha_j) + (1 - P_j) T_{N-1}(K,\beta)] \tag{16}$$

If the minimizing J turns out to be $J = M$, then

$$V_N(L,W,J) = \begin{bmatrix} \text{for } J = M\colon & 1 + P_M V_{N-1}(K,\alpha_M,M) + (1 - P_M) V_{N-1}(K,\beta,M) \\ \text{for } J \neq M\colon & P_M V_{N-1}(K,\alpha_M,J) + (1 - P_M) V_{N-1}(K,\beta,M) \end{bmatrix} \tag{17}$$

The quantity $S(L, K)$ represents any additional research and development expenditure which may be required by a choice of J; note that if K does not contain vehicles not contained in the combination L, then $S(L,K) = 0$. The calculation of $V_N(L,W,J)$ allows the introduction of any learning curve by making C_j a function of $V_N(L,W,J)$. The introduction of constraints, abort costs, and other complications is handled as before and will not change the foregoing outline for modifying the cost accounting method of the original model.

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